The Building of New Reverse Mortgage Payment Plans in the Korean Housing Market

Ma, Seungryul** · Cho, Deokho***

Abstract

A reverse mortgage system is recognized as an important financial vehicle to supplement current social security systems for elderly homeowners. This paper develops a new payment plan, called a graduated monthly payment which considers the living expenditures for the elderly and the consumer price index and analyzes two types of tenure plans: a constant monthly payment plan and a new graduated monthly plan based on different scenarios on the value of house and the longevity of borrowers in Korea. This paper evaluates the ratios of the monthly payments to the monthly living costs for households with heads 65 years of age and older through the reverse mortgage system. The objective is to determine a better payment system for elderly households in Korea which is a rapidly aging society. This would help elderly homeowners select a more appropriate payment type of reverse mortgages based on their life expectancy and living costs that can be applied to any other rapid aging country.

Keywords: Reverse mortgage system, tenure payment plan, constant and graduated payment programs.

I. Introduction

Korea has experienced a rapid increase in the number of elderly households. It has gradually approached the aging society like other developed countries, and as a result, the social security

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system has become a major social issue. In order to resolve this issue, Korea has utilized a government insured reverse mortgage system (Cho, et. al., 2004). This system utilizes a loan against a borrower’s home that does not need to be repaid as long as they live in their own housing. It is the reverse type of the payment pattern of the traditional forward mortgage. The system assists housing rich and cash-poor elderly homeowners to access additional income to meet living and medical expenses through the liquidation of their housing assets. Homeowners do not need a minimum income to qualify for a reverse mortgage. They can keep their homeownerships like a forward mortgage but are still responsible for paying their property taxes and homeowner insurance and for making property repairs.

There are three basic types of reverse mortgage systems, depending upon the payment method: tenure, term, and credit line plan. A tenure plan makes a monthly constant cash advance for the mortgagors as long as the borrowers live in their home. A term plan makes a monthly constant cash advance for a fixed period and a credit line plan makes payments up to the available principal limit whenever the borrower requests them (HUD, 2006). This study focuses on the tenure mortgage because it implements the life-time monthly payments for the elderly homeowners. However, this system pays a constant amount monthly to elderly households without considering changes in living expenses or the consumer price index. This paper develops a new payment method, called the graduated monthly payment, which considers the living costs of elderly homeowners as well as the consumer price index in order to reduce the risk of changes of financial market and living conditions. These two payment types are applied to the Korean housing market and the potential maximum levels of the two monthly payment types are analyzed. This new payment method will contribute to develop the actuarial model of reverse mortgage and also increase the feasibility of reverse mortgage programs in the Korean financial market because it reflects the real financial market circumstance.

In order to achieve these goals, the second section reviews the previous studies on the demand of the reverse mortgage system and notes its payment plans. The third section builds a graduated payment plan of tenure reverse mortgage, considering living expenditures and the consumer price index (CPI). The fourth section examines the stability of time series data and evaluates the profile, over the mortgagor’s life-time, of the percentage of ‘required expenditure’ provided by several different scenarios. The final section will suggest some policy implications of the new reverse mortgage payment plans.

II. Literature Review of the Reverse Mortgage Payment System

As the previous descriptions, several reverse mortgage systems have been developed. However, there is no alternative for securing the living costs for the life-time of households and
reducing the default risks. This paper focuses on the Home Equity Conversion Mortgage (HECM) insurance demonstration program which is a representative type of reverse mortgage systems insured by the federal government in the United States where the reverse mortgage is successfully implemented. It was created by the National Housing Act in 1987 (HUD, 2006) and designed to provide elderly homeowners with a financial vehicle to tap their housing equity without selling or moving from their homes. It allows a borrower aged 62 and older to borrow against the equity in a property that has limited outstanding debt. In October 1998, Congress altered the HECM into a permanent program which is secured by the government for the lenders and borrowers. As a result, the numbers of loans increased to 150,000 and a total of nearly 40,000 reverse mortgages have been insured under the HECM program since October 1999 (Rodda, et. al., 2000).

Rasmussen, et. al. (1995) asserted that both borrowers and lenders undertake risk in the process. Reverse mortgages pose three main sources of collateral risk meaning that the loan balance may grow to exceed the value of the collateral for lenders. The three risks are: 1) the borrower may live in the property so long that the continuing payments to the borrower exceed the value of the home; 2) interest rates may raise thereby increasing the interest payments and increasing the debt; 3) the property value may drop such that the value is less than expected when the loan becomes due. Moreover several scholars have noted the risks of the reverse mortgage system. Boehm and Ehrhardt (1992) asserted that the cash flows of a reverse mortgage mirrored the pattern of a life insurance policy in terms of the supporting living costs of the elderly. They (1994) developed a valuation model that quantified the interest rate risk inherent in fixed rate reverse mortgages and showed that the interest rate risk of reverse mortgages was greater than that of either a typical coupon bond or a regular mortgage. Therefore, a reverse mortgage should consider changes in future interest rates. Szymanoski, Jr. (1994) analyzed the risk included in reverse mortgage insurance and demonstrated how borrower longevity, interest rates and property value change all affected pricing.

However, HECM only provide mortgagors with a constant monthly payment in a tenure program irrespective of their longevity, living expenditure, and consumer price index, without considering the risk of reverse mortgages. A monthly loan advance is fixed in nominal terms. It has the potential to result in a mismatch between living expenditures and payment amount and also would result in less demand for the program if the inflation is predict (AARP, 2003). Therefore, this paper develops a new payment type, called a graduated monthly payment that takes into consideration the changes in living expenditures and the consumers’ price index in order to lessen the risks of reverse mortgage system in order to reflect the real situation of the Korean housing financial market and the demand of living expenditure of elderly.
III. Building of Payment Plan Models

1. The Measurement of Future Housing Values

The present value of housing price to the future certain time (t=N), which considers the monthly growth rate of housing price \( g_t \) and interest rate \( r_t \), can be calculated with this equation:

\[
PV_N = \frac{HP_N}{\prod_{t=1}^{N} (1 + r_t)} = HP_0 \prod_{t=1}^{N} \left(\frac{1 + g_t}{1 + r_t}\right)
\]  

\( PV_N \) = present value of a future housing price
\( HP_0 \) = current housing price (t=0)
\( HP_N \) = future value of housing price (t=N)
\( g_t \) = growth rate of housing price at time \( t \) (t=1,2,3,\ldots,N)
\( r_t \) = nominal interest rate at time \( t \) (t=1,2,3,\ldots,N)

The net discount ratio is presented as \( \left(\frac{1 + g_t}{1 + r_t}\right) \) in equation (1). If the time series of this net discount ratio is confirmed stationary, the mean value of the net discount ratio (NDR) can be used for calculating the present value of the future housing price (Haslag, et. al., 1991), Ma and Cho (2003)). And then equation (1) can be simplified like equation (2).

\[
PV_N = HP_0 \left(\frac{1 + g}{1 + r}\right)^N = HP_0 \cdot NDR^N
\]

\( PV_N \) = present value of future housing price
\( HP_0 \) = current housing price (t=0)
\( g \) = average growth rate of housing price
\( r \) = average nominal interest rate
\( NDR = \left(\frac{1 + g}{1 + r}\right) \) = mean value of net discount ratio

2. Determining the Maximum Level of Loan-to-Value (LTV)

This study determines the potential maximum level of LTV (Loan-to-Value) according to the figures of \( NDR^N \) in equation (2). LTV is calculated by multiplying the current housing price \( (HP_0) \) by the figures of \( NDR^N \). This study assumes that the required upfront costs (see note 3)
of a reverse mortgage are 5% of the current housing prices, based upon the study of Cho, Park, and Ma (2004). Ma and Cho (2003) showed empirically that the figures of $NDR^N$ were smaller than 1.0 and as the length of the maturity became longer, the figures of $NDR^N$ became smaller in the Korean financial market. This phenomenon eventually results from the fact that the mean value of the growth rate of the housing price is less than that of market interest rate in Korea.

3. Maximum Level of a Constant Monthly Payment

This study examines the maximum level of a constant monthly payment based on the maximum level of total payment amount ($LSUM^*$) over each period. It assumes the life-expectancies\(^1\) for the elderly as 20, 18, 15, and 12 years, based upon the life expectancy table and for the convenience of comparison. In a situation where the payments of an annuity are made at the beginning of interest conversion periods, this annuity is called annuity due (Muksian, 2003). The maximum level of constant monthly payments (annuities) can be calculated as equation (3) in annuity due.

$$a^* = \frac{LSUM^*}{\sum_{t=0}^{N-1} \left( \frac{1}{1+i} \right)^t} \tag{3}$$

$$a^* = \text{constant monthly payment}$$

$$LSUM^* = \text{maximum level of total payment amount}$$

$$\sum_{t=0}^{N-1} \left( \frac{1}{1+i} \right)^t = \text{cumulative discount factor (CDF)}$$

$$N = \text{remainder of borrower’s expected life}$$

$$i = \text{annuity rate}$$

After determining the maximum level of lump sum ($LSUM^*$), the maximum level of constant monthly payments ($a^*$) can be determined using the annuity rate. The specified level of annuity rate ($i$) can be decided through arrangements between lenders and borrowers.

4. Maximum Level of New Payment Type – Graduated Monthly Payment

This study develops and analyzes a new monthly payment method of reverse mortgage system- the graduated monthly payment that reflects the growth rate of living expenditures and
the growth rate of consumer prices. It is defined as follows: Provided that the time series of both
the growth rate of living expenditures and the growth rate of consumer prices are stationary
respectively, the starting amounts of the graduated monthly payment \((a)\) can be calculated
using the following relationship.

\[
LSUM^* = a \sum_{t=0}^{N-1} \left( \frac{1}{1+i} \right)^t = a \sum_{t=0}^{N-1} \left( \frac{1 + g_t}{1+i} \right)^t
\]

\(LSUM^*\) = maximum level of lump sum

\(PV\) = present value of future housing price

\[
a \sum_{t=0}^{N-1} \left( \frac{1}{1+i} \right)^t = \text{present value of constant monthly payment}
\]

\[
a \sum_{t=0}^{N-1} \left( \frac{1 + g_t}{1+i} \right)^t = \text{present value of graduated monthly payment}
\]

\(g_t\) = mean value of growth rate of expenditure \((g_{EXI})\), or consumer
price \((g_{CP})\)

Based upon this relationship, the level of starting amounts of graduated monthly payments
\((a)\) can be determined as in equation (5)

\[
a = \frac{LSUM^*}{\sum_{t=0}^{N-1} \left( \frac{1 + g_t}{1+i} \right)^t}
\]

\(a\) = base starting amount of graduated monthly payments

\(LSUM^*\) = maximum level of lump sum

\[
\sum_{t=0}^{N-1} \left( \frac{1 + g_t}{1+i} \right)^t = \text{cumulative discount factor (CDF)}
\]

Due to price inflation, the risk of decreasing purchasing power actually rises with the passage
of time. However, if a borrower selects graduated monthly payments linked either to price
inflation or required living expenditures, then the risk of suffering a shortfall in required
expenditure could be, in the former case, considerably reduced or in the latter case avoided
completely.
VI. Comparison between Constant Monthly Payment Plans and Graduated Monthly Ones

1. Data

This paper uses the monthly data from the apartment house\(^2\) price index (APT) of Kookmin Bank from January 1986 to December 2003 as a proxy variable of the housing price. The monthly data on after-tax yields on National Housing Bonds with 5-year maturity (HB5 (after tax))\(^3\) of the Korea National Statistical Office was used as a proxy variable of the market interest rate in the same period. The time series of the net discount ratio (NDR\(_t\)) is generated using the growth rate of APT price and the time series of HB5 and also the growth rate of APT by log differencing the time series of APT price. Figure 1 shows the time series of monthly growth rates of APT (G(APT)), HB5(after tax), and the net discount ratio (NDR\(_t\)) from January 1986 to December 2003.

![Figure 1: Monthly growth rates of APT (G(APT)), HB5(after tax), and the trend of NDR\(_t\) (01.1986 to 12.2003)](image)

In order to estimate the growth rates of living expenditures (\(g_{EXI}\)), the time series of monthly expenditures, (which is provided by the Korean National Statistical Office, for salary and wage earners’ households for the same period), was utilized. It also conducted seasonal adjustments to the time series of expenditures using the method of ratio to moving average. In order to obtain the growth rate of consumer prices (\(g_{CPI}\)), the time series of the quarterly consumer price index for the same period was utilized. Figure 2 shows the index of living expenditures (EXI) and consumer price index (CPI).
The time series of growth rate of expenditure ($g_{EXI}$) was generated by log differencing the seasonally adjusted time series of expenditure and the time series of growth rate of consumer price ($g_{CPI}$) was generated by the same way. Figure 3 shows the growth rate of expenditure ($g_{EXI}$) and the growth rate of consumer price ($g_{CPI}$).

2. The Methods of the Unit Root Test

This paper presents the test of the stationarity of time series. A time series is said to be stationary if the generating function for the series does not itself change through time. This
study conducts the unit root test in order to confirm the stationarity of time series. This study uses the methods of the ADF test (Augmented Dickey-Fuller test) and PP test (Phillips-Perron test) for testing the stationarity of the time series. ADF and PP tests use different methods to control for higher-order serial correlation in the series.

The test regression for the ADF test is as follows.

\[
\Delta y_i = \delta y_{i-1} + \alpha \sum_{i=1}^{m} \Delta y_{i-j} + \varepsilon_i \quad (6)
\]

\[
\Delta y_i = \beta_i + \delta y_{i-1} + \alpha \sum_{i=1}^{m} \Delta y_{i-j} + \varepsilon_i \quad (7)
\]

\[
\Delta y_i = \beta_0 + \beta_1 t + \delta y_{i-1} + \alpha \sum_{i=1}^{m} \Delta y_{i-j} + \varepsilon_i \quad (8)
\]

The ADF test makes a parametric correction for higher-order correlation by assuming that the \( y \) series follows an AR(p) process and adjusting the test methodology. The ADF approach controls higher-order correlation by adding lagged difference terms of the dependent variable \( y \) to the right-hand side of the regression. Phillips and Perron (1988) propose that the PP test is a unit root test method that can be used in a situation where the assumption on error term, \( u_t \), does not meet the condition of \( u_t \sim iid(0, \sigma_u^2) \). The test regression for the PP test is the AR(1) process. The OLS (ordinary least squares) estimation is primarily conducted with the DF model and modified t-statistics of \( \delta \), coefficient from the AR(1) regression, to account for the serial correlation in \( u_t \). If the time series of the net discount ratio, the growth rate of expenditure, and the growth rate of consumer price are confirmed stationary respectively, it can be concluded that the mean values of the time series can be used for calculating equation (2) to (5) in this research.

Table 1 reports the descriptive statistics of the growth rates of apartment prices (\( g_{APT} \)), housing bonds (HB5), and the time series of net discount ratio (NDR).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g_{APT} )</td>
<td>0.004991</td>
<td>0.013868</td>
</tr>
<tr>
<td>HB5(after tax)</td>
<td>0.007918</td>
<td>0.002385</td>
</tr>
<tr>
<td>net discount ratio</td>
<td>0.997105</td>
<td>0.014132</td>
</tr>
</tbody>
</table>

Note: \( g_{APT} \) = the growth rate of APT price

Table 2 reports the results of the unit root test of the methods of ADF test and PP test on the
growth rates of apartment prices \( g_{APT} \) and housing bonds (HB5).

### Table 2. The result of unit root test

<table>
<thead>
<tr>
<th>Variable</th>
<th>ADF statistic</th>
<th>PP statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g_{APT} )</td>
<td>-4.2518</td>
<td>-6.0747</td>
</tr>
<tr>
<td>HB5 (after tax)</td>
<td>-3.2157</td>
<td>-3.0081</td>
</tr>
</tbody>
</table>

Critical values: 1%: -4.0013, 5%: -3.4309, 10%: -3.1391

Notes. 1. Results from both Intercept and trend are included in the test equation.  
2. Null hypothesis: test series has a unit root.  
3. Critical values for rejection of hypothesis of a unit root are from MacKinnon.

The ADF and PP tests show that the time series of APT price \( g_{APT} \) reject the null hypothesis that there is a unit root at a 1% level of significance. But the time series of HB5 do not reject the null hypothesis at a 5% level of significance in ADF test and at a 10% level of significance in PP test. Therefore, the time series of the growth rate of APT price \( g_{APT} \) is stationary, but the time series of HB5 is nonstationary. And this analysis should determine whether the nonlinear expression \( \left( \frac{1 + g_t}{1 + r_t} \right) \) is itself a stationary series or not. Nonlinear transformations applied to nonstationary time series can yield stationary series (Haslag, et. al., 1991). As Table 3 indicates, the time series of the net discount ratio is stationary. This result notes that the mean value of the net discount ratio \( \left( \frac{1 + g}{1 + r} \right) \) can be used in calculating the present value of future housing prices.

### Table 3. Test for unit root

<table>
<thead>
<tr>
<th>Variable</th>
<th>ADF statistic</th>
<th>PP statistic</th>
<th>Critical values</th>
</tr>
</thead>
<tbody>
<tr>
<td>net discount ratio</td>
<td>-3.7676</td>
<td>-5.7410</td>
<td>1%: -3.4613, 5%: -2.8751, 10%: -2.5740</td>
</tr>
</tbody>
</table>

Notes. 1. Results from only Intercept are included in the test equation.

### 3. Determining the Maximum Level of Lump Sum Value

The maximum level of LTV (loan-to-value) can be determined according to the figures of \( NDR^N \) in equation (2). Based upon the mean value of the net discount ratio \( NDR = 0.997105 \) in Table 1, the values of \( NDR^N \) can be calculated and it is possible to evaluate the maximum level of LTV as shown in Table 4. If it is assumed that the current housing value \( HP_c \) is 200 million Won (1$ = about 1,000 Won) and the upfront costs\(^4\) of a reverse mortgage loan are 5%
of the current housing value \( (HP_0 \times 5\% ) \), the maximum values of lump sum \( (LSUM^*) \) are shown in Table 4.

### Table 4. The maximum level of lump sum and loan-to-value ration (unit: Korean Won(₩))

<table>
<thead>
<tr>
<th>The remainder of borrower's expected life (= N)</th>
<th>N=240 (20 yrs)</th>
<th>N=216 (18 yrs)</th>
<th>N=180 (15 yrs)</th>
<th>N=120 (10 yrs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LTV</td>
<td>0.50</td>
<td>0.53</td>
<td>0.59</td>
<td>0.71</td>
</tr>
<tr>
<td>( LSUM^* )</td>
<td>94,747,590</td>
<td>101,574,970</td>
<td>112,749,470</td>
<td>134,171,690</td>
</tr>
</tbody>
</table>

1. Table assumes that current housing value \( (HP_0) \) is 200 million Won \( ($1 = \text{about} \ 1,000 \) \).
2. It assumes that upfront costs are 5% of housing value because borrowers finance all loan costs from a reverse mortgage loan without paying contract costs.

#### 1) Determining the Maximum Level of Constant Monthly Payments

After determining the value of \( LSUM^* \), the maximum level of constant monthly payments can be determined by using equation (3). Ma and Park (2004) showed that the time series of the life insurers’ ratio of total asset profits (RTAP) was stationary and the mean value of life insurers’ RTAP was approximately 7% per annum in Korea. This ratio is applied to equation (3) as the annuity rate and Table 5 reports the monthly cumulative discount factor (CDF) using \((7/12)\% \) as the monthly annuity rate. The maximum level of constant monthly payments \( (a^* ) \) can be determined using the value of CDF as shown in Table 5.

### Table 5. Cumulative discount factor (CDF) and maximum value of constant monthly advance (unit: ₩)

<table>
<thead>
<tr>
<th>N=240 (20 yrs)</th>
<th>N=216 (18 yrs)</th>
<th>N=180 (15 yrs)</th>
<th>N=120 (10 yrs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDF</td>
<td>129,7349</td>
<td>123,3391</td>
<td>111,9050</td>
</tr>
<tr>
<td>( a^* )</td>
<td>730,320</td>
<td>823,540</td>
<td>1,007,550</td>
</tr>
</tbody>
</table>

Note: 1. \( \text{CDF} = \sum_{t=0}^{N-1} \left( \frac{1}{1+i} \right)^t \), where, \( i=0.07/12 \)

#### 2) Determining the Maximum Level of Graduated Monthly Payments

This study analyzes the graduated monthly payments that reflect the growth rate of expenditures and consumer prices, respectively. The maximum level of base starting amounts
of graduated monthly payments is determined using equation (5). Table 6 shows the result of the unit root test on these time series from the first quarter 1986 to the last quarter 2003.

<table>
<thead>
<tr>
<th>Variables</th>
<th>ADF statistics</th>
<th>PP statistics</th>
<th>Critical values</th>
</tr>
</thead>
<tbody>
<tr>
<td>EXI(growth)</td>
<td>-6.6737</td>
<td>-6.6805</td>
<td>1%: -3.5270, 5%: -2.9036,</td>
</tr>
<tr>
<td>CPI(growth)</td>
<td>-7.2905</td>
<td>-7.3077</td>
<td>10%: -2.5892</td>
</tr>
</tbody>
</table>

Note: Results from only Intercept are included in the test equation.

It is shown that the statistics of both ADF test and PP test reject the null hypothesis that there is a unit root at 1% level of significance in Table 6. This analysis notes that the two time series are stationary, and that the mean value of these variables in evaluating the maximum level of base starting amounts \( a \) of graduated monthly payments is shown in equation (5).

<table>
<thead>
<tr>
<th>Variables</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Mean( per year)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EXI(growth)</td>
<td>0.026131</td>
<td>0.036209</td>
<td>10.45%</td>
</tr>
<tr>
<td>CPI(growth)</td>
<td>0.011494</td>
<td>0.009613</td>
<td>4.60%</td>
</tr>
</tbody>
</table>

Table 7 shows that the mean value of the growth rate of expenditures \( g_{EXI} \) is 0.026131 and that of consumer prices \( g_{CPI} \) is 0.011494. It shows that the mean value of growth rate of expenditures is more than two times that of consumer prices during the last 18 years (the first quarter 1986 – the last quarter 2003) in Korea. Table 8 shows the monthly cumulative discount factors (CDF), where 7% is the annuity rate \( i=0.07 \) and the mean values of growth rates are 

\[
g_{EXI}=0.026131 \quad \text{and} \quad g_{CPI}=0.011494
\]

respectively. The maximum level of base starting amounts \( a_{EXI} \) and \( a_{CPI} \) of graduated monthly payments can be determined using the value of 

\[
CDF_{EXI} \quad \text{and} \quad CDF_{CPI}.
\]

<table>
<thead>
<tr>
<th>The remainder of borrower's expected life (= N)</th>
<th>N=240 (20 yrs)</th>
<th>N=216 (18 yrs)</th>
<th>N=180 (15 yrs)</th>
<th>N=120 (10 yrs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( CDF_{EXI} )</td>
<td>261.0899</td>
<td>239.9777</td>
<td>191.679</td>
<td>125.1035</td>
</tr>
<tr>
<td></td>
<td>$CDF_{\text{EXI}}$</td>
<td>$a_{\text{EXI}}$</td>
<td>$CDF_{\text{CPI}}$</td>
<td>$a_{\text{CPI}}$</td>
</tr>
<tr>
<td>----------</td>
<td>---------------------</td>
<td>------------------</td>
<td>--------------------</td>
<td>-----------------</td>
</tr>
<tr>
<td></td>
<td>172.2986</td>
<td>362,890</td>
<td>160.0204</td>
<td>423,270</td>
</tr>
<tr>
<td></td>
<td>139.8956</td>
<td>588,220</td>
<td>101.2254</td>
<td>805,950</td>
</tr>
<tr>
<td></td>
<td>101.2254</td>
<td>1,072,490</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>549,900</td>
<td>1,325,470</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes. 1. $CDF_{\text{EXI}} = \sum_{t=0}^{N-1} \left( \frac{1 + g_{\text{EXI}}}{1 + i} \right)^t$; where, $g_{\text{EXI}} = 0.026131/4, \ i = 0.07/12$

2. $CDF_{\text{CPI}} = \sum_{t=0}^{N-1} \left( \frac{1 + g_{\text{CPI}}}{1 + i} \right)^t$; where, $g_{\text{CPI}} = 0.011494/4, \ i = 0.07/12$

3. Table assumes that current housing value ($H_P$) is 200million won.

4. It assumes that upfront costs are 5% of current housing price.

5. $a_{\text{EXI}} = \frac{PV}{CDF_{\text{EXI}}}$; The base starting amount of using the value of $g_{\text{EXI}}$

6. $a_{\text{CPI}} = \frac{PV}{CDF_{\text{CPI}}}$; The base starting amount of using the value of $g_{\text{CPI}}$

3. Comparison of Constant and Graduated Payments

Figure 4 shows the maximum level of constant monthly payments and the base starting amounts of graduated monthly payments. The level of base starting amount using the value of $g_{\text{EXI}}$ is the lowest and the level of constant monthly payment is the highest among these three payment methods.

![Figure 4. Comparison of each payment method](image-url)
Table 9 shows the ratios to constant monthly payments ($a_{CON} = 1.0$).

### Table 9. Ratios to constant monthly payments ($a_{CON} = 1.0$)

<table>
<thead>
<tr>
<th>The remainder of borrower’s expected life (= N)</th>
<th>N=240 (20 yrs)</th>
<th>N=216 (18 yrs)</th>
<th>N=180 (15 yrs)</th>
<th>N=120 (10 yrs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{EXI}$</td>
<td>0.50</td>
<td>0.51</td>
<td>0.58</td>
<td>0.69</td>
</tr>
<tr>
<td>$a_{CPI}$</td>
<td>0.75</td>
<td>0.77</td>
<td>0.80</td>
<td>0.86</td>
</tr>
<tr>
<td>$a_{CON}$</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

In the case where the remainder of the borrower’s life expectancy is 240 months, Table 9 shows that the level of constant monthly payments is two times as large as that of base starting amount in the case of using the value of $g_{EXI}$ and 1.33 times as large as that of base starting amount in the case of using the value of $g_{CPI}$. However, in the graduated monthly payment methods, the payments grow over time. Figure 5 shows the trends of monthly payments to 216 months.

![Figure 5. Trends of monthly advances of each payment method](image-url)

It shows that the amount of monthly payments of the graduated type become higher than that of constant monthly payment after the 120th month. Table 10 reports monthly income and expenditure per household by the age of household head. According to the analysis of Seog and Kim (2000), the appropriate level of monthly expenditure of 60 years of age and over is 0.78 times as large as that of 50-59 years of age in Korea.
Table 10. Income and expenditure per household by the age of household head (unit: ₩)

<table>
<thead>
<tr>
<th>Age of household head</th>
<th>50-59 years : (A)</th>
<th>60 years and over : (B)</th>
<th>(B)/(A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monthly Income</td>
<td>2,171,000</td>
<td>1,508,000</td>
<td>0.69</td>
</tr>
<tr>
<td>Monthly Expenditure</td>
<td>1,663,000</td>
<td>1,297,000</td>
<td>0.78</td>
</tr>
</tbody>
</table>

Note 1. The number in parenthesis is the appropriate level of expenditure (that is the mean value of required monthly expenditure) on the basis of Seog and Kim (2000)

Table 11 reports the ratios of graduated payments using the value of $g_{EXI}$ to required expenditure (required expenditure = 1.0) that are classified according to the level of current housing price ($HP_0$). For instance, if $HP_0$ is 200million won and life expectancy is 216 months, the level of graduated payments using the value of $g_{EXI}$ only covers 27% of the required expenditure; if $HP_0$ is 600million won, it covers 80% of required expenditure. If $HP_0=200$million won and N=120, the level of graduated payments using the value of $g_{EXI}$ charges 68% of required expenditure and if $HP_0=600$million won, it is 204% of required expenditure.

Table 11. Ratios of graduated payments using the value of $g_{EXI}$ to required expenditure

<table>
<thead>
<tr>
<th>The remainder of borrower's expected life (= N)</th>
<th>N=240 (20 yrs)</th>
<th>N=216 (18 yrs)</th>
<th>N=180 (15 yrs)</th>
<th>N=120 (10 yrs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>required expenditure</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>$RE_0$=200million won</td>
<td>0.23</td>
<td>0.27</td>
<td>0.37</td>
<td>0.68</td>
</tr>
<tr>
<td>$RE_0$=400million won</td>
<td>0.46</td>
<td>0.54</td>
<td>0.74</td>
<td>1.36</td>
</tr>
<tr>
<td>$RE_0$=600million won</td>
<td>0.69</td>
<td>0.80</td>
<td>1.12</td>
<td>2.04</td>
</tr>
</tbody>
</table>

In the case of graduated payments using the value of $g_{EXI}$, the level of each ratio to required expenditure in Table 11 is constant over time ($t=1,2,3,\ldots,N$) because the growth rate of monthly payments covers that of living expenditures. However the ratios of constant payments to required expenditures will decline over time because it does not reflect the growth rate of expenditures. The ratios of graduated payments using the value of $g_{EXI}$ to required expenditure is similar to the case of constant payments though it is not equal in size because the value of $g_{CPI}$ is smaller than that of $g_{EXI}$.
Table 12 shows that the ratios of constant payments to required expenditures decline over time. For instance, if $HP_0=200$ million won and $N=216$, the level of constant payments at $t=1$ is 52% of required expenditure, that at $t=216$ it only covers 13% of required expenditure. These figures are considerably higher than the case of graduated payments using the value of $g_{EXI}$ (see Table 11). If $HP_0=600$ million won and $N=216$, then at $t=1$ it covers 156% of required expenditures, at $t=216$ it represents 39% of required expenditures. These results show that in the case of using the value $g_{EXI}$, the constant payment is beneficial to the borrower who owns relatively the lower cost housing but the graduated payment is to the higher price homeowners even though ratios to the required expenditure decline over time.

Table 12. Ratios to required expenditure (required expenditure = 1.0): constant payments

<table>
<thead>
<tr>
<th>Remainder of borrower’s Expected life</th>
<th>Housing Value (million₩)</th>
<th>Required Expenditure</th>
<th>Ratios to required expenditure with the passage of time</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$T=1$</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>1.00</td>
<td>0.46</td>
</tr>
<tr>
<td>N=240 (20 yrs)</td>
<td>400</td>
<td>1.00</td>
<td>0.92</td>
</tr>
<tr>
<td></td>
<td>600</td>
<td>1.00</td>
<td>1.38</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>1.00</td>
<td>0.52</td>
</tr>
<tr>
<td>N=216 (18 yrs)</td>
<td>400</td>
<td>1.00</td>
<td>1.04</td>
</tr>
<tr>
<td></td>
<td>600</td>
<td>1.00</td>
<td>1.56</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>1.00</td>
<td>0.64</td>
</tr>
<tr>
<td>N=180 (15 yrs)</td>
<td>400</td>
<td>1.00</td>
<td>1.28</td>
</tr>
<tr>
<td></td>
<td>600</td>
<td>1.00</td>
<td>1.92</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>1.00</td>
<td>0.98</td>
</tr>
<tr>
<td>N=120 (10 yrs)</td>
<td>400</td>
<td>1.00</td>
<td>1.96</td>
</tr>
<tr>
<td></td>
<td>600</td>
<td>1.00</td>
<td>2.94</td>
</tr>
</tbody>
</table>

Table 13 presents the ratios of current levels of graduated payments that reflect consumer price ($g_{CPI}$) to the level of required expenditure and the dynamic trends of these ratios through time.

Table 13. Ratios to required expenditure (required expenditure = 1.0): graduated payments using the value of $g_{CPI}$
It shows that the method of graduated payments using the value of $g_{CPI}$ reflects relatively high levels of ratios in its early stages, but declines through time as expected. For instance, if $HP_0=200$ million won and $N=216$, in case of 65 years old, the level of graduated payments of using the value of $g_{CPI}$ at $t=1$ is 40% of required expenditure and if $HP_0=600$ million won and $N=216$, the level of graduated payments using the value of $g_{CPI}$ at $t=1$ is 120% of required expenditure. These figures are higher than the case of graduated payments of using the value of $g_{EXI}$ but lower than the case of constant payments. Meanwhile, if $HP_0=200$ million won and $N=216$, the level of graduated payments of using the value of $g_{CPI}$ at $t=216$ is 18% of required expenditure and if $HP_0=600$ million won and $N=216$, the level of graduated payments of using the value of $g_{CPI}$ at $t=216$ is 54% of required expenditure. These figures are lower than the case of graduated payments using the value of $g_{EXI}$ but higher than the case of constant payments.

In summary, the starting amount of payments for constant payments is higher than that of a graduated payment. However, the gap becomes smaller over time and is eliminated after 100 months. After that point, the graduated payment is higher than the constant payment. Therefore, a graduated payment system would be better than a constant payment system because the elderly need more money for health care and home stay costs as they get older (Stucki, 2005). It also reduces the risks of housing financial market because it reflects the changes of housing price and interest rate. Comparing the results of Table 11, 12, 13, elderly homeowners could select more appropriate levels of reverse mortgages and payment methods according to their ages (remainder of borrower’s expected life), the current values of their homes, savings, other

<table>
<thead>
<tr>
<th>expected life</th>
<th>(million ₩)</th>
<th>$t=1$</th>
<th>$T=24$</th>
<th>$t=120$</th>
<th>$t=180$</th>
<th>$t=216$</th>
<th>$t=240$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N=240$ (20 yrs)</td>
<td>200</td>
<td>1.00</td>
<td>0.35</td>
<td>0.32</td>
<td>0.23</td>
<td>0.18</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>1.00</td>
<td>0.70</td>
<td>0.64</td>
<td>0.46</td>
<td>0.36</td>
<td>0.32</td>
</tr>
<tr>
<td></td>
<td>600</td>
<td>1.00</td>
<td>1.05</td>
<td>0.96</td>
<td>0.69</td>
<td>0.54</td>
<td>0.48</td>
</tr>
<tr>
<td>$N=216$ (18 yrs)</td>
<td>200</td>
<td>1.00</td>
<td>0.40</td>
<td>0.37</td>
<td>0.26</td>
<td>0.21</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>1.00</td>
<td>0.80</td>
<td>0.74</td>
<td>0.52</td>
<td>0.42</td>
<td>0.36</td>
</tr>
<tr>
<td></td>
<td>600</td>
<td>1.00</td>
<td>1.20</td>
<td>1.11</td>
<td>0.78</td>
<td>0.63</td>
<td>0.54</td>
</tr>
<tr>
<td>$N=180$ (15 yrs)</td>
<td>200</td>
<td>1.00</td>
<td>0.51</td>
<td>0.47</td>
<td>0.33</td>
<td>0.27</td>
<td></td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>1.00</td>
<td>1.02</td>
<td>0.94</td>
<td>0.66</td>
<td>0.54</td>
<td></td>
</tr>
<tr>
<td></td>
<td>600</td>
<td>1.00</td>
<td>1.53</td>
<td>1.41</td>
<td>0.99</td>
<td>0.81</td>
<td></td>
</tr>
<tr>
<td>$N=120$ (10 yrs)</td>
<td>200</td>
<td>1.00</td>
<td>0.84</td>
<td>0.77</td>
<td>0.54</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>1.00</td>
<td>1.68</td>
<td>1.54</td>
<td>1.08</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>600</td>
<td>1.00</td>
<td>2.52</td>
<td>2.31</td>
<td>1.62</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
potential social security benefits, and individual preferences because a graduated payment system can provide the elderly with several different programs.

V. Conclusions

Reverse mortgages have several positive effects for the elderly as the previous discussions. Seniors do not have to move or lose the title over their most important asset but the reverse mortgages only allow borrowers to tap a proportion of their housing equity. It also supports the spouses after the borrowers die. Moreover the left-over assets can be transferred to their heirs after paying off the loan.

This study contributes to two aspects on the reverse mortgages. At first this paper theoretically developed a new payment plan, called a gradual payment method in the reverse mortgage system- that considers the changes in living costs and consumer prices even through a graduated payment method is available in the forward mortgage system. The paper also analyzes the payment ratios to the required expenditure of two payment methods, a constant payment which is the payment system of HECM in the United States and a graduated one that uses the data on four different life expectancy groups and three different housing values.

Second, this study empirically examined the effects of two different reverse mortgage systems in order to meet living expenditures and to improve the quality of life for elderly homeowners in the Korean housing financial market. This paper focused on the analysis of the appropriate methods of tenure payments for reverse mortgages. The ratios of monthly living costs to a constant payment type and a graduated payment type would help elderly homeowners select more appropriate payment methods of reverse mortgages according to their needs and their housing values.

This paper also provides more information about risk, discount rates, changes in housing values and the consumer price index. A graduated payment system can alleviate the risk of decreasing the purchasing power on the reverse mortgage system due to long-term financial market instability. This program can be applied to the housing finance market of any other rapid aging country as well as the Korean housing finance market where a reverse mortgage system will be introduced eventually. It also is suitable for countries with unstable financial markets because it reflects the changes of living cost and the housing financial market. These results suggest a new payment type that considers the living costs of mortgagors and changes in future housing prices and provides appropriate payment alternatives based on the longevity of mortgagors and housing value.

Uncertainty on future health and long-term case expenses will increase the demand of reverse mortgage by tapping the housing assets without moving out or sell their homes. High level of
homeownership and home equity can provide the seniors with an important opportunity to stay their own housing without moving out by using the reverse mortgages. These funds can be used for the long-term care funds in home. Consequently the reverse mortgages can be utilized as one of the most important policy alternative for the elderly welfare. However this study only focused on the simulation of several different reverse mortgages because the reverse mortgage system is not introduced in the Korean housing financial market. Therefore, the more specific studies on the each different aspect of reverse mortgage programs are needed after its introduction in the Korean housing market.

**Notes**

1. Life expectancies of females are as follows; 63 years of age: 20.13 years, 65 years of age: 18.43 years, 69 years of age: 15.18 years, 77 years of age: 9.62 years (Sources: 2001 Life tables, National Statistical Office, Korea).
2. Apartment housing is the most popular type which is not rental housing but similar with the condominium and a house price index is a nation-wide official data which is released by the Korean government. This data is surveyed by Kookmin Bank in order to determine the transaction prices of ownership housing and lump sum deposit rental housing. The starting year of this survey is 1986 and its sample size is about 19,000 observations.
3. There are several bonds such as Korea Securities Depositary, National Housing Bond, and Corporation Bond in Korea. However, National Housing Bond is a representative proxy variable of the market interest rate in the housing market.
4. An upfront cost is composed of the origination fee (2%), upfront mortgage insurance premium (2%), and third-party closing costs and etc. This paper assumes that total upfront costs are 5% of housing values.
5. There is no a life expectancy data on households. Therefore this paper uses an average life expectancy data of females as a proxy variable of that of households because females live longer than males. This paper use 218 months (18 years) as a criteria year for comparing several different programs.
5. There are not data on the living expenditure of age 65 years old elderly people. Therefore the data on the age 60 is used as a proxy variable.

**References**


Stucki, Barbara R. (2005), Use Your Home to Stay at Home: Expanding the Use of Reverse Mortgages for Long-term Care: A Blueprint for Action, the National Council on the Aging.
