Abstract

I develop a life cycle model trying to explain this fact that owner-occupied housing is usually larger than rental housing in the data. The model relies on three crucial assumptions to deliver the predictions that matches the facts in the data. The assumptions are: idiosyncratic income shocks, higher adjustment costs for owner-occupied housing than rental housing and a down payment constraints. The model simulations confirms the intuition that the size for rental housing is much smaller than that of owner-occupied housing given plausible calibrated parameter values. Moreover, this model also predicts we tend to see more large rental housing in high income volatility area than in low income volatility area. This may helps to understand the cross-metropolitan area difference of size distribution of houses.

1 Introduction

In typical housing markets, the houses for rent are smaller than owner-occupied houses. The average size of rental housing is about 1145 square feet, while the average size for owner-occupied housing is about 2184 square feet, almost two times large (AHS 2005 National). Moreover, the size distribution of rental housing and owner-occupied housing are very different. Figure-1 is a plot of the kernel density of size distribution of rental
housing and owner-occupied housing. At the high end of the distribution, there is much fewer large rental housing than large owner-occupied housing, or there is just no larger enough rental housing. In addition to the fact that rental housing is smaller than owner-occupied housing, the size of rental housing caries across metropolitan areas, especially at the high end of the distribution. Table-1 gives the size quantile of rental housing in 13 MSAs that are covered by 2004 American Housing Survey Metropolitan Sample. The mean size of rental housing ranges from 952 square feet in Denver to 1338 square feet in Atlanta, while the 99% quantile ranges from 2500 square feet in Oklahoma to 17719 square feet in Atlanta. This paper tries to address the basic question why rental housing is smaller than owner-occupied housing and also provides one explanation of the cross-sectional differences of size distribution of rental housing.

Owner-occupied housing is preferred to rental housing either because of tax advantage associated with owner-occupied housing or because of moral hazard problem associated with rental housing. In a frictionless complete market, households would strictly prefer owner-occupied housing. Then why households choose rental housing? A simple answer to this question is that households are borrowing constrained or there is down-payment requirement for buying a house. Households with insufficient wealth accumulation cannot afford the down payment thus have to rent. But this answer is not perfectly satisfactory, constrained households can buy smaller houses, but this is not what we see in the data. Why constrained households do not buy smaller houses instead of renting? Some papers in the literature (Cocco 200, Silos 2005) implicitly assuming that there is no market supply for small houses by simply putting an lower bound on the house size available. But why market is not willing to supply small houses? This paper offers an answer from the demand side of the market instead of supply side by arguing that when households buy, they buy large houses; when they rent, they rent smaller ones.

Typical households start with very little wealth and very low income. Thus young households cannot afford to either buy a big house or rent a big apartment because of
binding borrowing constraint. The options available are either to buy a smaller houses
or rent a smaller apartment. If there are no other frictions, the household would choose
to buy a smaller house because of owner-occupied house is typically preferred to rental
house. But in housing market, transactions costs associated with owner-occupied housing
is much larger than rental housing. Young households, whose income are typically rising
over time, would prefer to adjust their housing consumption with the rising income and
wealth accumulation frequently. Thus young households choose to rent small apartments
instead of own because of high transactions costs associate with owner-occupied housing.
When households become older, they accumulate wealth over time and receive higher
income, they are less constrained eventually, and are able to smooth their housing con-
sumption. Therefore households adjust their housing consumption less frequently when
they are older, wealthier and less constrained. They then choose to buy own-occupied
housing because of the advantage of owner-occupied housing outweighed the high trans-
actions costs. The owner occupied housing is also typically larger because households
are richer when they buy. The cross-sectional difference of size distribution can also be
explained by the above story. The income volatilities in different metropolitan area are
different. Different income volatility causes different tenure choice decision, thus the size
distribution of rental houses. For example, higher income volatility delays the house pur-
chasing, then there are more richer households renting, thus more larger rental houses,
which pushes up the size distribution of rental housing, and vise versa.

Based on the above story, there are three key ingredients of the model I propose
here: 1) The rising age earning profile. Households are constrained initially and adjust
their housing consumption more frequently because of the rising age earning profile.
Also households become less constrained and wealthier when they become older. 2)
Borrowing constraints or down payment requirement. This makes households unable to
borrow against their future income and buy large owner-occupied housing to smooth
their housing consumption when they are young. 3) Higher transactions costs associated
with owner-occupied housing, which makes young households rent instead of own.
This paper uses a life cycle model to operationalize the above story. Although there are no research on answering the same question we raised here, there are a large amount of literature studying life cycle models with housing. Li and Yao (2006) is the closest to our model. They calibrated a life cycle model to study the effect of price change on portfolio choice and welfare. Their model has all the elements we have in this paper; further more, they also have stochastic house price and stock market. Cocco (2005) also uses a life cycle model to study the portfolio choice in the presence of housing, but there is no rental market and a lower bound of the house size is exogenously set as mentioned before his model. Both Li and Yao (2006) and Cocco (2005) are not equilibrium models. However, there are other literature using general equilibrium model with housing to study some traditional macroeconomic problems. Just to name a few. Yang (2005) explains the housing consumption over life cycle using a general equilibrium Overlapping-Generation model. Silos (2006) studies the portfolio choice and business cycle properties of an economy with housing. Different from Yang (2005), Silos (2006) has aggregated uncertainty. Nakajima (2005) studies the impact of earning instability on portfolio choice and house prices. Fernandez and Krueger (2005) uses a model with durable and non-durable consumption goods to study the life cycle profiles of consumptions. All the above equilibrium models have no rental market. Silos (2005), however develops an equilibrium model with rental market and he claims that such a model can account for the observed wealth distribution of US economy.

The life cycle model in this paper is much simpler. First it is not an equilibrium model, although implicit in the model I assume that the market is in steady state. Second there are no price fluctuation in this model because of the implicit assumption of steady state. Third, there are no stock market in this model, which stems from the convention in the incomplete market literature.

The rest of the paper is organized as follows. Section II lays out the model elements. Section III provides a simple discussion of the choice of parameter values of the model.
Section IV gives the main numerical results. Section V concludes.

2 The Model

Consider a life cycle model with only income uncertainty. Housing has dual role: it is one of the consumption goods and it can be used as the only collateral against which households can borrow. Trading of housing incurs large transactions cost. Households enter the model at age 20, and start working and receiving labor income. They retire at age 65 and receive no labor income thereafter. Households die at age 80.

2.1 Demographics

The economy is populated by a continuum of \textit{ex ante} identical households. All households start their working life at age 25 with no financial asset and no house. When they work, they receive labor income which is independent across households but highly autocorrelated over time. The households trade in the asset market and housing market each period and consume a numeraire consumption good and housing, either rental housing or owner-occupied housing. Household retire at age $RT$ and receive no labor income after retirement. Households die at $T$, and leave all unused wealth as a bequest. Each modeling period is one year in this model.

2.2 Preference

Households derive utility from consumption of a numeraire good, $C$ and from the service flow of housing, $H$ and the bequests transferred to their children upon death. The service flow of housing is assumed to proportional to the housing stock. Thus the utility function can be defined over housing stock directly. The utility function is assumed to be time separable with constant discount factor $\beta$, and of the constant relative-risk
aversion (CRRA) class. i.e.

\[ \sum_{t=0}^{T} \beta_t g(C_t, \delta H_t)^{1-\gamma} \frac{1}{1-\gamma} \]  \hspace{1cm} (1)

where \( g(C_t, H_t) \) is a CES aggregator, i.e.

\[ g(C_t, H_t) = \left( C_t^{\varepsilon/(\varepsilon-1)} + \psi H_t^{\varepsilon/(\varepsilon-1)} \right)^{(\varepsilon-1)/\varepsilon} \]  \hspace{1cm} (2)

Where \( \delta = 1 \) if the individual lives in a rental housing, \( \delta = \eta > 1 \) if he lives in an owner-occupied housing. This is simply to say that individuals strictly prefer owner-occupied housing, this assumption can be justified either by moral hazard argument or tax advantage argument, which are both well documented in tenure choice literature. Here I just take this reduced form approach. Note this assumption is crucial to the results, because for rich enough individuals, they would eventually buy a house because they are not borrowing constrained as discussed later in this section.

\( \gamma \) here is the elasticity of intertemporal substitution and \( \gamma > 1 \). \( \varepsilon \) is the elasticity of intratemporal substitution. If \( \varepsilon = 1 \), the \( g \) function becomes Cobb-Douglas. \( \psi \) is just a weight between numeraire consumption and housing consumption.

The utility function from bequest is given by:

\[ \phi(b) = K \frac{b^{1-\gamma}}{1-\gamma} \]  \hspace{1cm} (3)

The constant \( K \) reflects the parents’ concern about leaving bequest to their children.

2.3 Endowment

Each household is endowed a stream of income over their working subject to idiosyncratic shocks.

\[ \log y_t = f_t + u_t + v_t \]  \hspace{1cm} (4)
where \( f(t) \) is the deterministic age earning profile. \( u_t \) is a persistent shock to the income process, which follows the following AR(1) process and independent across households.

\[
\log u_{t+1} = \rho \log u_t + \epsilon_{t+1}
\]  (5)

\( \epsilon \) is assumed to be i.i.d normal with mean 0, and variance \( \sigma^2_\epsilon \).

\( v_t \) is a transitory shock to income, distributed as normal with mean \( -\frac{1}{2} \sigma^2_v \), and variance \( \sigma^2_v \). the transitory shock is i.i.d over time and across households. The mean of the shock is picked so that whenever I change the variance of the process, the mean value of the income does not change. This is designed to isolate the effect of variance rather than the level of income.

This income risk is idiosyncratic and cannot be diversified away. The idiosyncrasy typically would cause precautionary saving as long as the marginal utility is convex and thus reduces individuals’ ability to smooth housing consumption, and thus one driving force of housing adjustment.

### 2.4 Market

The only financial asset is the risk free bond with interest rate \( r \). For individuals who live in rental houses, they are not allowed to borrow, i.e. their financial asset holdings must be nonnegative,while for owners, they can borrow up to a fixed fraction of the house value (typically the fraction would be one minus the down payment requirement). The borrowing constraint makes a typical young household unable to buy a house or rent a large house even he anticipates higher future income and wealth. Thus young households need to change their housing consumption frequently to match their income profile instead of smooth their housing consumption over time in this incomplete market.
For simplicity, I assume that the house price $P$ and house rent $R$ are constant, and $R < rP$. This assumption makes buying a house even more preferable than renting. More realistic assumption, for example, stochastic price and rent, though fits the facts better, won’t change the basic qualitative prediction of the model. One thing it can change is that stochastic price and rent may make individuals more reluctant to adjust their housing consumption following classical real options arguments. Given that this won’t change the qualitative prediction of the model, I adopt this simplifying assumption to reduce the dimensionality of the dynamic programming problem. But this assumption is still justifiable because this economy has no aggregate uncertainty by law of large numbers, thus in steady states, the prices are constant.

2.5 Borrowing Constraint

I assume that only collateralized borrowing is allowed in this economy. Interest rates for borrowing and lending are equal. This is to say that mortgage and deposits are perfect substitutes. I use $B_t$ to denote the net financial asset holding. For renters, borrowing is not allowed, i.e.

$$B_t \geq 0, \text{ for renters}$$

Home-owners can borrow up to a fixed fraction $\alpha$ of the total house value. i.e.

$$B_t \geq -\alpha PH_t, \text{ for owners}$$

Here $1 - \alpha$ is the minimum down payment requirement.
2.6 Transactions Costs

Due to the heterogeneity and spatial fixity of housing, both potential buyers and sellers in the housing market are forced to spend large amount time and resource in searching, as a consequence, the associated explicit and implicit transactions costs are very high. Compared with owner-occupied housing, searching for rental housing and moving between rental housing incur much smaller costs. While for owner-occupied housing, this large transactions costs includes opportunity cost of market search, brokerage fee, origination fee, moving costs and psychological costs. Some of these costs are proportional to the house price, such as brokerage fee and origination fee, others are not. Given these stylized fact, I assume that transactions costs of moving between and moving out of rental houses are 0, while the transactions costs associated with moving between and out of owner-occupied housing are given by two parts, one fixed costs and one variable costs that is proportional to the selling price of the house.

\[ c = c_1 + c_2 PH \]

2.7 The Household’s Problem

The household’s problem can be formulated recursively. In this problem, the renter’s state variable are given by \((B, u, v, t)\), while the state variable space must include housing for home owners, i.e. the state variables for home-owners are \((B, u, v, H, t)\). Where \(B\) denotes the net asset holding, \(y\) is the endowed labor income this period, \(t\) is age, \(H\) is housing. Let \(V^0(B, y, t)\) denote the value function for renter, and \(V^1(B, H, u, v, t)\) the value function for owner. According to the demographic transitions, there are four cases depending on in which stage the household is living.
2.7.1 Working Period

From age 25 to age 65, the household works and receives labor income. Households choose tenure choice, housing consumption, numeraire consumption and asset holding each period. Renters choose between remaining as a renter or becoming a home-owner.

\[
V^0(B, y, t) = \max \begin{cases} 
\max_{\{C'_0, H'_0, B'_0\}} \left\{ U(C'_0, H'_0) + \beta E[V^0(B', y', t + 1)|u] \right\}, \\
\max_{\{C''_0, H''_0, B''_0\}} \left\{ \eta U(C''_0, H''_0) + \beta E[V^1(B''_0, H''_0, y', t + 1)|u] \right\}
\end{cases} 
\]

\[s.t.: \]
\[C'_0 + RH'_0 + B'_0 = B(1 + r) + y\]  \hspace{1cm} (6)
\[C'_0 > 0, H'_0 > 0\]  \hspace{1cm} (7)
\[B'_0 \geq 0\]  \hspace{1cm} (8)

\[C''_0 + PH''_0 + B''_0 = B(1 + r) + y\]  \hspace{1cm} (9)
\[C''_0 > 0, H''_0 > 0\]  \hspace{1cm} (10)
\[B''_0 \geq -\alpha PH''_0\]  \hspace{1cm} (11)

Home-owners choose between remaining in the current housing, moving to another house or downgrading and becoming a renter.
\[ V^1(B, H, y, t) = \max \begin{cases} \max_{\{C'_1, H'_1, B'_1\}} \left\{ U(C'_1, H'_1) + \beta E [V^0(B'_1, y', t + 1)|y]\right\}, \\
\max_{\{C''_1, H''_1, B''_1\}} \left\{ U(C''_1, H''_1) + \beta E [V^1(B''_1, H''_1, y', t + 1)|y]\right\} \end{cases} \]

\[ s.t. \]
\[ C'_1 + R H'_1 + B'_1 = B(1 + r) + y + (1 - c_2) PH - c_1 \] (13)
\[ C'_1 > 0, \ H'_1 > 0 \] (14)
\[ B'_1 \geq 0 \] (15)
\[ C''_1 + P H''_1 + B''_1 = B(1 + r) + y + PH - (c_1 + c_2 PH) \ell \{H \neq H''_1\} \]
\[ B''_1 \geq -\alpha PH''_1 \] (16)
\[ C''_1 > 0, \ H''_1 > 0 \] (17)

### 2.7.2 Retirement Period

After retirement, households don’t receive any labor income. Thus renter’s problem can be formulated as:

\[ V^0(B, t) = \max \begin{cases} \max_{\{C'_0, H'_0, B'_0\}} \left\{ U(C'_0, H'_0) + \beta V^0(B'_0, t + 1)\right\}, \\
\max_{\{C''_0, H''_0, B''_0\}} \left\{ U(C''_0, H''_0) + \beta V^1(B''_0, H''_0, t + 1)\right\} \end{cases} \]

\[ s.t. \]
\[ C'_0 + R H'_0 + B'_0 = B(1 + r) \] (19)
\[ C'_0 > 0, \ H'_0 > 0 \] (20)
\[ B'_0 \geq 0 \] (21)
\[ C''_0 + P H''_0 + B''_0 = B(1 + r) \]
\[ C''_0 > 0, \ H''_0 > 0 \] (22)
\[ B''_0 \geq -\alpha PH''_0 \] (23)
and home-owner’s problem can be formulated as:

\[
V^1(B, H, t) = \max \left\{ \begin{array}{c}
\max_{\{C'_1, H'_1, B'_1\}} \{ U(C'_1, H'_1) + \beta V^0(B'_1, t + 1) \}, \\
\max_{\{C''_1, H''_1, B''_1\}} \{ U(C''_1, \eta H''_1) + \beta V^1(B''_1, H''_1, t + 1) \}
\end{array} \right\}
\]

(24)

\[
s.t. \\
C'_1 + RH'_1 + B'_1 = B(1 + r) + (1 - c_2)PH - c_1
\]

(25)

\[
C'_1 > 0, \ H'_1 > 0
\]

(26)

\[
B'_1 \geq 0
\]

(27)

\[
C''_1 + PH''_1 + B''_1 = B(1 + r) + PH - (c_1 + c_2 PH)\ell\{H \neq H''_1\}
\]

(28)

\[
C''_1 > 0, \ H''_1 > 0
\]

(29)

2.7.3 Last Period

In the last period, household decides consumption and bequest. For renter:

\[
V^0(B, t) = \max \left\{ \begin{array}{c}
\max_{\{C'_0, H'_0, b\}} \{ U(C'_0, H'_0) + \beta \phi(b'_0) \}, \\
\max_{\{C''_0, H''_0, b\}} \{ U(C''_0, \eta H''_0) + \beta \phi(b''_0) \}
\end{array} \right\}
\]

(30)

\[
s.t. \\
C'_0 + RH'_0 + b = B(1 + r)
\]

(31)

\[
C'_0 > 0, \ H'_0 > 0
\]

(32)

\[
b'_0 > 0
\]

(33)

\[
b''_0 = B(1 + r) - C''_0 - c_1 - c_2 PH''_0
\]

(34)

\[
C''_0 > 0, \ H''_0 > 0
\]

(35)
For home-owner:

\[
V^1(B, H, t) = \max \left \{ \begin{array}{c}
\max_{(C'_1, H'_1, B'_1)} \left \{ U(C'_1, H'_1) + \beta \phi(b'_0) \right \}, \\
\max_{(C''_1, H''_1, B''_1)} \left \{ U(C''_1, \eta H''_1) + \beta \phi(b''_0) \right \}
\end{array} \right \}
\]

\text{s.t.}

\[ C'_1 + RH'_1 + b'_1 = B(1 + r) + (1 - c_2)PH - c_1 \]  
\[ C'_1 > 0, \ H'_1 > 0 \]  
\[ b'_1 > 0 \]  
\[ b''_1 = B(1 + r) - C''_1 - (c_1 + c_2 PH)\ell \{ H \neq H''_1 \} - (c_1 + c_2 PH''_1) \]  
\[ C''_1 > 0, \ H''_1 > 0 \]  
\[ b''_1 > 0 \]  

\( \ell \{ H \neq H''_1 \} \) is an indicator function, such that

\[ \ell \{ H \neq H''_1 \} = \begin{cases} 
1 & \text{if } H \neq H''_1 \\
0 & \text{otherwise}
\end{cases} \]

There exists no analytical solution to the above Bellman equation, thus I use numerical method to solve the problem in next section.

3 Calibration

3.1 Demographics

One model period is corresponding to one year. Individuals are assumed to live for 65 periods, i.e. \( T = 60 \). One can think of an individual starts working at real age of 20 and dies at age of 85. The retirement age is set at 65, i.e. \( RT = 45 \).
3.2 Preference

The value of the discount factor is chosen to be 0.95, this follows most literature on precautionary saving. The intratemporal elasticity of substitution is chosen to be $\varepsilon = 1.07$, because in Piazzesi et al (2006), they show that the intratemporal elasticity should be slightly greater than 1 and in their calibration, they also adopted 1.07. $\Psi$ is chosen to be 0.23. This is chosen such that for renters, the housing expenditure share is about 0.2, which is the housing expenditure share in the data. The intertemporal elasticity is chosen to be 3, as most literature suggests that risk aversion should be between 2-5. $\eta$ is set to be 1.1, which is chosen to match the home ownership rate of 69%.

3.3 Income Process

The autocorrelation $\rho$ is set to 0.935 following the vast literature. The standard deviation of the persistent shock $\sigma_\varepsilon$ is set to 0.5. The standard deviation of the transitory shock $\sigma_v$ is set to 0.3 in the base case as in Nakajima (2005). The persistent shock AR(1) process is then discretized using Tauchen’s method. The age earning profile is taken from Hugget (1996), which is interpolated from Hansen (1993).

3.4 House Price and Rent

To reduce the computational burden, I assume that house price and rent is deterministic and constant. This is not a realistic assumption, but in this model, what is really important is the price to rent ratio, rather than the dynamics. So this unrealistic assumption won’t change the qualitative prediction of the model. The house price is set to be 8, which produces an average price-to-income ratio or 4.3. The rent is set to be 0.7, which gives a price-rent ratio of 1.14.
3.5 Other Parameters

The interest rate is set to be 5%, and the down payment requirement is set to be 20%, which means the required loan-to-value ratio $\alpha$ is set to 0.8. The fixed transactions costs is set to be 0.2, which is about about 2% of house value and the proportional transactions costs is set to be 3%, which is the prevailing commission fee in the real estate brokerage industry. Table-2 summarizes the calibrated base case parameter values.

4 Results

4.1 Typical Household Behavior

I first show the life cycle profile of a typical household to illustrate the mechanism of the model. Figure-2 displays the housing tenure and house size over the life cycle of a typical household. It shows exactly the pattern as described in the introduction. When the household is young, he lives in rental housing, and the house size changes every year due to income fluctuations. When the household is older, he chooses to own a house, and then very few adjustments are made after that. (For this household, there is only one adjustment after that actually, for other households. In the simulated data, there are households who make no adjustments of owner-occupied housing and there are also some households who make two adjustments. But there is no household who makes more than 3 adjustments among the 10000 households I simulated). As a consequence of rising income and the above housing tenure choice decision, size of the rental housing for this household is smaller than size of the owner-occupied house.

Figure-3 shows the asset holding of the same household over the life cycle. The household is initially borrowing constrained, the asset holding is kept at 0 for quite a time. Then he accumulates wealth gradually over time until he is not constrained any
more. At that time, the households is able to smooth his housing consumption, thus he buys a house and the asset holding jumps down almost to the lower limit constrained by house value. He then again gradually build up his asset holding to prepare for retirement. After retirement, since there are no uncertainties any more, the asset holding decreases smoothly towards the end of his life. Again, figure-3 confirms our intuition described in the introduction.

4.2 House Size

To get a cross-sectional presentation of the total population, I simulate 10000 households for each age. Then compute the homeownership rate, house size given the simulated cross-section. For the base case parameters, the average size of owner-occupied housing is 2.00, while the average size for rental housing is only 0.51, which is only about one fourth of the size of owner-occupied housing. Moreover, the maximum size of owner-occupied housing is 12.00, while the maximum size for rental housing is only 3.60, which is again only slightly more than one fourth of the maximum of owner-occupied housing. These qualitative results match the stylized facts in the housing market, where average size of owner-occupied housing is larger than rental housing, and very large rental housing is not available in the market, which reflects the fact that households do not have demand for large rental housing. Notice that here the difference between the sizes of rental housing and owner-occupied housing are much bigger than in the data. This may explain that in the data, the difference between rental housing and owner-occupied housing is multi-dimensional, while the model only considers the size dimension. In the data, the quality of rental housing is typically lower than that of the owner-occupied housing. For example, the lot size of owner-occupied housing (83787 sf) is much large than that of rental housing (60177 sf). The average rating of the unit as a place to live of rental housing is 7.58, which is lower than that of the owner-occupied housing 8.94. The rating of the neighborhood as a place to live of rental housing (6.61) is also lower than that of the owner-occupied
housing (8.26). Thus the size difference generated from the model may be interpreted as a composite of size and quality. Figure-4 and figure-5 compare the size distribution of the actual data and data simulated from the model for rental housing and owner-occupied housing respectively. They show that the model can largely match the size distribution in the data.

One of the key predictions of the model is that size of rental houses increases with income volatility. In this model, however, there are two effects of increasing income volatility. The first effect is of course increased income uncertainty. But cross-sectionally, increasing volatility also increases the cross-section income dispersion. On the one hand, uncertainty effect makes the households unwilling to upgrade to owner-occupied housing early in their lives, thus when they buy a house, they have accumulated more wealth, thus both larger owner-occupied housing and rental housing. On the other hand, the dispersion effect makes the households in the bottom part of the income distribution even poorer, thus makes their rental housing even smaller, which may attenuate the uncertainty effect if only average size is used. Thus to illustrate how income volatility affects house size, I compute different quantiles of rental housing size and size of owner-occupied housing for different volatility parameter values. Table-3 is a summary of that result. The volatility measure used in table-2 is the standard deviation of the transitory shock.

As in table-3, the size in the bottom quantiles of rental housing decreases with the volatility measure, in which case the dispersion effect of volatility dominates. While the size in the top quantiles increases with the volatility measure, in which case the uncertainty effect dominates. This observation is only true for rental housing. Because the dispersion effect on owner-occupied housing is not clear. Our focus in this paper is on the uncertainty effect on size of rental housing, especially the size in the top quantile of the distribution. Panel A of table-3 shows that the size of top quantile increases with volatility. This means that as income becomes more volatile, we tend to see more large
rental housing. For example, if volatility equals 0.1, the size of 95% quantile is only 0.89, while the 95% quantile for the case of 0.4 volatility is 1.66, which almost double the size of low volatility case. Another observation is that in the market of low volatility (0.1), the largest rental housing is 3.36. While the largest size for very high volatility (0.4) market is 4.48. These results may help to understand why in some metropolitan area, there are lots of large rental apartment; while only small rental apartments are available in other metropolitan area.

### 4.3 Homeownership Rate

The base case parameters also replicates the actual homeownership rate, the model predicts a homeownership rate of 69.07%. while the actual homeownership rate is 69.00% in the fourth quarter of 2005. Figure-6 depicts another aspects of homeownership rate, the homeownership rate for different age households. The horizontal axis is the model age of households, i.e. 0 means household of age 20. This figure matches the fact in the data that homeownership rate increases sharply in the early life of the households, then remains flat until very late in their life. But it fails to account for the facts that the homeownership rate for any age never reaches 1 and the homeownership rate for very young households is also not zero in the data. The reason for the failure is that the wealth inequality produced from the model is low relative to the wealth inequality in the data, which is the problem for most life cycle model in the literature. The resolution of this problem often requires either very unrealistic dispersed income or additional assumption on entrepreneurship.

Figure-7 plots the comparative statics of cross-sectional homeownership rate with respect to income volatility. Contrary to the results in early literature, the homeownership rate does not decrease monotonically as a function of income volatility. The reason for this result is again because of the two dimensional effects of increasing volatility. On the one hand, higher volatility means higher income uncertainty for individuals, thus reduces
homeownership rate; on the other hand, higher volatility also leads to higher income dispersion, which enables more households to overcome the borrowing constraint and buy houses. As a consequence of this, the empirical test of the effect of income volatility on homeownership rate has to take into consideration

4.4 Average life Cycle Patterns

This section explores the model implication of life cycle patterns of housing consumption and asset holdings to see whether the model delivers the features in the data. This serves as a robustness check of how well the model works as compared to the facts in the data. Figure-8 shows the average housing consumption over the life cycle. We see that housing consumption increases early in the life and slow downsizing later in life. The housing consumption peaks at actual age of 65, which coincides with the retirement age. This pattern matches qualitatively the pattern we see in the data. Figure-9 shows the evolution of wealth composition over the life cycle. Both financial wealth and total wealth (defined as financial wealth+housing wealth) are hump-shaped. Households do not hold much wealth when they are young because of low income and consumption smoothing motive. They borrow as much as possible to buy houses and save in the form of housing assets. As time goes by, households start to increase financial asset holding. Both financial wealth and total wealth peak at age of 64, the year before retirement. After retirement, households start to dissave as there are no other income after retirement in this model. The wealth accumulated before retirement is then used to finance consumption later in life. Our model reproduces important facts about life cycle composition of wealth: young households do not save in financial assets, but rather in housing, as households become older, financial assets become an important part of the households’ wealth portfolio. In summary, these life cycle patterns show that our model is in line with the some important facts in the data, which implies that our model assumptions and calibrations are not unrealistic.
4.5 Other Comparative Statics

This section explores how house size changes as some of the parameter values change. First, we study the effect of borrowing constraint. The main effect of change $\alpha$ from 0.8 to 0.75, which means a tighter borrowing constraint, is increase of both rental house size and the size of owner-occupied houses. The average rental house size increases from 0.51 to 0.54, while the average size of owner-occupied housing increases from 2.00 to 2.05. Tighter borrowing constraint delays households’ purchasing decision. Therefore when they buy, they have higher income and more wealth. Thus the size of both rental housing and owner-occupied housing increase.

If transaction costs of adjusting owner-occupied housing is higher, households buy house later, which again implies larger house size for both rental housing and owner-occupied housing. For example, an increase of proportional transaction costs from 3% to 3.5%, leads to an increase of rental house size from 0.51 to 0.56, and the size of owner occupied housing from 2.00 to 2.05.

A higher persistence parameter $\rho$ makes households willing to buy their houses earlier, because of higher persistence means less fluctuating income, thus less fluctuating housing consumption. Then Households cares less about housing adjustment needs because of income fluctuation. Therefore, high persistence parameter leads to smaller house size, both rental and owner-occupied housing. For example, an increase of $\rho$ from 0.9 to 0.92 leads to a decrease of rental house size from 0.51 to 0.49 and owner-occupied housing from 2.00 to 1.98.

Increased elasticity of substitution $\varepsilon$ has two effects for the base case parameter values. First it increases the expenditure share of housing consumption, second it delays the housing buying decision. These two effects offsetting each other. Thus for small range of elasticity of substitution value, the results does not change much. For example, an
increase of the intertemporal elasticity of substitution from 1.07 to 1.17, leads to a rental house size decrease from 0.51 to 0.507. While the size of owner-occupied housing decreases from 2.00 to 1.995.

5 Conclusion

Motivated by the fact that owner-occupied housing is normally larger than rental housing in the data, I developed a life cycle model trying to explain this fact. The model relies on three crucial assumptions to deliver the predictions that matches the facts in the data. The assumptions are: idiosyncratic income shocks, higher adjustments for owner-occupied housing than rental housing and a down payment constraints. The simulated results confirms the intuition that given plausible calibrated parameter values, the size for rental housing is much smaller than that of owner-occupied housing. Moreover, this model also predicts we tend to see more large rental housing in high income volatility area than in low income volatility area. This may helps to understand the cross-metropolitan area difference of size distribution of houses.
References


Journal of Economic Dynamics & Control


Table 1: Size Quantile of Rental Housing (AHS 2004 Metropolitan)

<table>
<thead>
<tr>
<th>MSA</th>
<th>1%</th>
<th>5%</th>
<th>10%</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
<th>90%</th>
<th>95%</th>
<th>99%</th>
<th>Mean</th>
<th>Std</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atlanta</td>
<td>375</td>
<td>550</td>
<td>700</td>
<td>800</td>
<td>1035</td>
<td>1300</td>
<td>1800</td>
<td>2100</td>
<td>17719</td>
<td>1338</td>
<td>1960</td>
</tr>
<tr>
<td>Cleveland</td>
<td>105</td>
<td>400</td>
<td>500</td>
<td>665</td>
<td>900</td>
<td>1200</td>
<td>1900</td>
<td>2400</td>
<td>1787</td>
<td>1185</td>
<td>1434</td>
</tr>
<tr>
<td>Denver</td>
<td>300</td>
<td>450</td>
<td>550</td>
<td>700</td>
<td>900</td>
<td>1200</td>
<td>1800</td>
<td>2200</td>
<td>3600</td>
<td>952</td>
<td>659</td>
</tr>
<tr>
<td>Hartford</td>
<td>99</td>
<td>320</td>
<td>400</td>
<td>600</td>
<td>800</td>
<td>1100</td>
<td>1500</td>
<td>1825</td>
<td>3060</td>
<td>956</td>
<td>966</td>
</tr>
<tr>
<td>Indianapolis</td>
<td>200</td>
<td>480</td>
<td>576</td>
<td>741</td>
<td>900</td>
<td>1200</td>
<td>1600</td>
<td>1950</td>
<td>4200</td>
<td>1086</td>
<td>994</td>
</tr>
<tr>
<td>Memphis</td>
<td>400</td>
<td>533</td>
<td>600</td>
<td>772</td>
<td>970</td>
<td>1200</td>
<td>1600</td>
<td>2000</td>
<td>3400</td>
<td>1086</td>
<td>647</td>
</tr>
<tr>
<td>New Orleans</td>
<td>99</td>
<td>322</td>
<td>500</td>
<td>700</td>
<td>920</td>
<td>1200</td>
<td>1800</td>
<td>2300</td>
<td>11958</td>
<td>1245</td>
<td>1639</td>
</tr>
<tr>
<td>Oklahoma</td>
<td>400</td>
<td>520</td>
<td>600</td>
<td>750</td>
<td>919</td>
<td>1200</td>
<td>1500</td>
<td>1800</td>
<td>2500</td>
<td>1019</td>
<td>458</td>
</tr>
<tr>
<td>Pittsburgh</td>
<td>130</td>
<td>344</td>
<td>400</td>
<td>600</td>
<td>800</td>
<td>1200</td>
<td>2000</td>
<td>2700</td>
<td>12901</td>
<td>1159</td>
<td>1462</td>
</tr>
<tr>
<td>Sacramento</td>
<td>250</td>
<td>500</td>
<td>600</td>
<td>720</td>
<td>925</td>
<td>1200</td>
<td>1600</td>
<td>1850</td>
<td>2800</td>
<td>1054</td>
<td>925</td>
</tr>
<tr>
<td>Santa Louis</td>
<td>99</td>
<td>430</td>
<td>510</td>
<td>720</td>
<td>900</td>
<td>1124</td>
<td>1600</td>
<td>2000</td>
<td>4200</td>
<td>1075</td>
<td>1045</td>
</tr>
<tr>
<td>San Antonio</td>
<td>200</td>
<td>440</td>
<td>500</td>
<td>650</td>
<td>900</td>
<td>1200</td>
<td>1600</td>
<td>1950</td>
<td>8832</td>
<td>1049</td>
<td>931</td>
</tr>
<tr>
<td>Seattle</td>
<td>240</td>
<td>400</td>
<td>500</td>
<td>683</td>
<td>880</td>
<td>1100</td>
<td>1500</td>
<td>2000</td>
<td>3500</td>
<td>1051</td>
<td>1015</td>
</tr>
</tbody>
</table>
Table-2 Summary of Base Case Parameter Values

<table>
<thead>
<tr>
<th>Preference</th>
<th>$\beta$</th>
<th>0.935</th>
<th>Income Process</th>
<th>$\mu$</th>
<th>0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>3</td>
<td></td>
<td></td>
<td>$\rho$</td>
<td>0.9</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>1.07</td>
<td></td>
<td></td>
<td>$\sigma_\varepsilon$</td>
<td>0.5</td>
</tr>
<tr>
<td>$\Psi$</td>
<td>0.23</td>
<td></td>
<td></td>
<td>$\sigma_v$</td>
<td>0.3</td>
</tr>
<tr>
<td>$\eta$</td>
<td>1.1</td>
<td>House Rent</td>
<td></td>
<td>$R$</td>
<td>0.7</td>
</tr>
<tr>
<td>Demographics</td>
<td>$T$</td>
<td>60</td>
<td>House Price</td>
<td>$P$</td>
<td>8</td>
</tr>
<tr>
<td>$RT$</td>
<td>45</td>
<td></td>
<td>Fixed Transactions Costs</td>
<td>$c_1$</td>
<td>0.2</td>
</tr>
<tr>
<td>LTV</td>
<td>$\alpha$</td>
<td>0.8</td>
<td>Proportional Transactions Costs</td>
<td>$c_2$</td>
<td>0.03</td>
</tr>
<tr>
<td>Interest Rate</td>
<td>$r$</td>
<td>3%</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 3: House Size in Different Quantiles

Panel A: Size Quantile of Rental Housing

<table>
<thead>
<tr>
<th>Volatility</th>
<th>5%</th>
<th>10%</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
<th>90%</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.24</td>
<td>0.26</td>
<td>0.31</td>
<td>0.47</td>
<td>0.58</td>
<td>0.66</td>
<td>0.89</td>
</tr>
<tr>
<td>0.2</td>
<td>0.22</td>
<td>0.25</td>
<td>0.32</td>
<td>0.49</td>
<td>0.63</td>
<td>0.84</td>
<td>1.15</td>
</tr>
<tr>
<td>0.3</td>
<td>0.18</td>
<td>0.24</td>
<td>0.34</td>
<td>0.52</td>
<td>0.68</td>
<td>1.02</td>
<td>1.34</td>
</tr>
<tr>
<td>0.4</td>
<td>0.16</td>
<td>0.23</td>
<td>0.35</td>
<td>0.54</td>
<td>0.73</td>
<td>1.28</td>
<td>1.66</td>
</tr>
</tbody>
</table>

Panel B: Size Quantile of Owner-Occupied Housing

<table>
<thead>
<tr>
<th>Volatility</th>
<th>5%</th>
<th>10%</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
<th>90%</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.64</td>
<td>0.65</td>
<td>0.68</td>
<td>1.20</td>
<td>2.08</td>
<td>2.72</td>
<td>5.80</td>
</tr>
<tr>
<td>0.2</td>
<td>0.65</td>
<td>0.68</td>
<td>0.80</td>
<td>1.52</td>
<td>2.72</td>
<td>3.03</td>
<td>6.48</td>
</tr>
<tr>
<td>0.3</td>
<td>0.71</td>
<td>0.73</td>
<td>0.81</td>
<td>1.62</td>
<td>2.87</td>
<td>3.34</td>
<td>7.31</td>
</tr>
<tr>
<td>0.4</td>
<td>0.77</td>
<td>0.79</td>
<td>0.83</td>
<td>1.72</td>
<td>3.19</td>
<td>3.67</td>
<td>7.85</td>
</tr>
</tbody>
</table>
Figure 1: Kernel Density of House Size Distribution

Figure 2: Housing Tenure and Size Over Life Cycle
Figure-3 Asset Holding Over Life Cycle

Figure-4 Kernel Density of Size Distribution or Rental Housing
Figure-5 Kernel Density of Size Distribution of Owner-Occupied Housing

Figure-6 Home Ownership Rate Over Life Cycle
Figure-7 The Effect of Volatility on Homeownership Rate

Figure-8 Housing Consumption Over Life Cycle
Figure 9: Average Household Wealth over Life Cycle

- Financial Wealth
- Total Wealth